

SYSTEMS OF LINEAR EQUATIONS

Two lines on an xy -coordinate grid are called a system of linear equations. They intersect at a point unless they are parallel or the equations are different forms of the same line. The point of intersection is the only pair of (x, y) values that will make *both* equations true. One way to find the point of intersection is to graph the two lines. However, graphing is both time-consuming and, in many cases, not exact, because the result may only be a close approximation of the coordinates.

When two linear equations are written equal to y (in general, in the form $y = mx + b$), we can take advantage of the fact that both y values are the same (equal) at the point of intersection. For example, if two lines are described by the equations $y = -2x + 5$ and $y = x - 1$, and we know that both y values are equal, then the other two sides of the equations must also be equal to each other. We say that both right sides of these equations have “equal values” at the point of intersection and write $-2x + 5 = x - 1$, so that the result looks like the work we did with equation mats.

We can solve this equation in the usual way and find that $x = 2$. Now we know the x -coordinate of the point of intersection. Since this value will be the same in both of the original equations at the point of intersection, we can substitute $x = 2$ in either equation to solve for y : $y = -2(2) + 5$ so $y = 1$ or $y = 2 - 1$ and $y = 1$. So the two lines in this example intersect at $(2, 1)$.

For additional information, see the Math Notes boxes in Lessons 5.2.2, 5.2.3, and 5.2.4 of the *Core Connections, Course 3* text.

Example 1

Find the point of intersection for $y = 5x + 1$ and $y = -3x - 15$.

Substitute the equal parts of the equations.

$$5x + 1 = -3x - 15$$

Solve for x .

$$8x = -16$$

$$x = -2$$

Replace x with -2 in either original equation and solve for y .

$$y = 5(-2) + 1$$

$$y = -10 + 1$$

$$y = -9$$

$$y = -3(-2) - 15$$

$$y = 6 - 15$$

$$y = -9$$

The two lines intersect at $(-2, -9)$.

Example 2

The Mathematical Amusement Park is different from other amusement parks. Visitors encounter their first decision involving math when they pay their entrance fee. They have a choice between two plans. With Plan 1 they pay \$5 to enter the park and \$3 for each ride. With Plan 2 they pay \$12 to enter the park and \$2 for each ride. For what number of rides will the plans cost the same amount?

The first step in the solution is to write an equation that describes the total cost of each plan. In this example, let x equal the number of rides and y be the total cost. Then the equation to represent Plan 1 for x rides is $y = 5 + 3x$. Similarly, the equation representing Plan 2 for x rides is $y = 12 + 2x$.

We know that if the two plans cost the same, then the y value of $y = 5 + 3x$ and $y = 12 + 2x$ must be the same. The next step is to write one equation using x , then solve for x .

$$5 + 3x = 12 + 2x$$

$$5 + x = 12$$

$$x = 7$$

Use the value of x to find y .

$$y = 5 + 3(7) = 26$$

The solution is $(7, 26)$. This means that if you go on 7 rides, both plans will have the same cost of \$26.

Problems

Find the point of intersection (x, y) for each system of linear equations.

1. $y = x - 6$
 $y = 12 - x$

2. $y = 3x - 5$
 $y = x + 3$

3. $y = 2x + 16$
 $y = 5x + 4$

4. $y = 3x - 5$
 $y = 2x + 14$

5. $y = x + 7$
 $y = 4x - 5$

6. $y = 7 - 3x$
 $y = 2x - 8$

Write a system of linear equations for each problem and use them to find a solution.

7. Jacques will wash the windows of a house for \$15.00 plus \$1.00 per window. Ray will wash them for \$5.00 plus \$2.00 per window. Let x be the number of windows and y be the total charge for washing them. Write an equation that represents how much each person charges to wash windows. Solve the system of equations and explain what the solution means and when it would be most economical to use each window washer.
8. Elle has moved to Hawksbluff for one year and wants to join a health club. She has narrowed her choices to two places: Thigh Hopes and ABSolutely fABulus. Thigh Hopes charges a fee of \$95 to join and an additional \$15 per month. ABSolutely fABulus charges a fee of \$125 to join and a monthly fee of \$12. Write two equations that represent each club's charges. What do your variables represent? Solve the system of equations and tell when the costs will be the same. Elle will only live there for one year, so which club will be less expensive?
9. Misha and Nora want to buy season passes for a ski lift but neither of them has the \$225 needed to purchase a pass. Nora decides to get a job that pays \$6.25 per hour. She has nothing saved right now but she can work four hours each week. Misha already has \$80 and plans to save \$15 of her weekly allowance. Who will be able to purchase a pass first?
10. Ginny is raising pumpkins to enter a contest to see who can grow the heaviest pumpkin. Her best pumpkin weighs 22 pounds and is growing at the rate of 2.5 pounds per week. Martha planted her pumpkins late. Her best pumpkin weighs 10 pounds but she expects it to grow 4 pounds per week. Assuming that their pumpkins grow at these rates, in how many weeks will their pumpkins weigh the same? How much will they weigh? If the contest ends in seven weeks, who will have the heavier pumpkin at that time?
11. Larry and his sister, Betty, are saving money to buy their own laptop computers. Larry has \$215 and can save \$35 each week. Betty has \$380 and can save \$20 each week. When will Larry and Betty have the same amount of money?

Answers

1. (9, 3)

2. (4, 7)

3. (4, 24)

4. (19, 52)

5. (4, 11)

6. (3, -2)

7. Let x = number of windows, y = cost. Jacques: $y = 15 + 1x$; Roy: $y = 5 + 2x$. The solution is (10, 25), which means that the cost to wash 10 windows is \$25. For fewer than 10 windows use Roy; for more than 10 windows, use Jacques.

8. Let x = weeks, y = total charges. Thigh Hopes: $y = 95 + 15x$; ABSolutely fABulus: $y = 125 + 12x$. The solution is (10, 245). At 10 months the cost at either club is \$245. For 12 months use ABSolutely fABulus.

9. Let x = weeks, y = total savings. Misha: $y = 15x + 80$; Nora: $y = 25x$. The solution is (8, 200). Both of them will have \$200 in 8 weeks, so Nora will have \$225 in 9 weeks and be able to purchase the lift pass first. An alternative solution is to write both equations, then substitute 225 for y in each equation and solve for x . In this case, Nora can buy a ticket in 9 weeks, Misha in 9.67 weeks.

10. Let x = weeks and y = weight of the pumpkin. Ginny: $y = 2.5x + 22$; Martha: $y = 4x + 10$. The solution is (8, 42), so their pumpkins will weigh 42 pounds in 8 weeks. Ginny would win (39.5 pounds to 38 pounds for Martha).

11. Let x = weeks, y = total money saved. Larry: $y = 35x + 215$; Betty: $y = 20x + 380$. The solution is (11, 600). They will both have \$600 in 11 weeks.